



## Abstract

Timing methods and performance metrics are important in the heavily industrialized world we live in. Industrial plants use metrics to measure quality of production, help make decisions, and drive the strategy of the organization. However, there are many factors to be considered when measuring performance based on a metric; of which we will be analyzing the importance of product variation. We will be analyzing assembly line timings, whilst controlling for product variance, to show accounting for product variance improves one's ability to predict machine times. In addition, we will be analyzing the current "statistical" methods used by an industrial partner and comparing it to a new method we propose. The data will be analyzed with statistical methods such as: ANOVA, ANCOVA, multiple linear regression, and others, with most of the calculations being done with the statistical software, R.

## Background Information

Currently, Pella has the following metrics:

Cycle Time

- The time (in seconds) between the start of one unit to the start of the next

Lowest Repeatable Cycle Time (LRCT)

- The average between the two cycle times that are the closest and lowest by time
- So for the set of cycle times {5,7,8,10,11,15}, the LRCT would be 7.5

## Objectives

- Examine the current Pella metrics
  - Mainly LRCT
- Identify if product variation is important when estimating machine timing
- Make a statistical model that will predict machine time given product qualities

## Methodology

We started by collecting data on the production floor from a machine that wraps windows in Saran wrap to prepare for shipping. We collected the following data on the process: the length and width of the windows, the machine wrapping times, whether the window is contained in cardboard or not, and the time in which the window was handled by the worker.

In order to test whether the different product qualities had a significant effect on the machine time, we created a linear model with the predictors: Width  $X_1$ , Length  $X_2$ , and Cardboard vs Plain windows  $X_3$ ; using the R software.

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

Once we had this linear model, we were able to test whether each of the predictors was significant when predicting machine time, i.e if that product quality has an effect on the machine timing. This was done by finding the t statistic for each predictor: and finding the probability of achieving a t statistic as large or larger, i.e. if the product quality had no effect on machine times.

$$t_i = \frac{\hat{\beta}_i}{\hat{\sigma}_i}$$

Now that we knew which product qualities affected machine timings, we tried to find the best combination to include into our final linear model. This was done by running an ANOVA test on our full model containing length, width, and the presence of cardboard. Once we had found our final model we calculated the adjusted  $R^2$

$$R_{adjusted}^2 = 1 - (1 - R^2) \frac{n - 1}{n - p - 1}$$

After creating our final model, we were able to evaluate the current metric (LRCT) Pella is using. Using the LRCT for our dataset we found the expected time it would take to complete all 60 windows, considering and not considering whether or not they were covered in cardboard. We did this to evaluate if the LRCT could better predict machine time, if it accounted for the windows' covering.

## Results

After fitting the data to a linear model with all the predictors we get the following:

|                     | $\beta_i$ | Standard Error | T-Value | Pr(> t ) |
|---------------------|-----------|----------------|---------|----------|
| (Intercept)         | 6.7480    | 8.3189         | 0.81    | 0.4207   |
| Width ( $X_1$ )     | 0.7224    | 0.1641         | 4.40    | 0.0000   |
| Length ( $X_2$ )    | 0.2507    | 0.1193         | 2.10    | 0.0402   |
| Cardboard ( $X_3$ ) | 33.0006   | 4.4943         | 7.34    | 0.0000   |

This regression table shows that all of the product qualities are significant when predicting machine time and gives the  $\beta_i$ 's for each quality. The adjusted  $R^2$  for this model is 0.776 and the ANOVA analysis tells us that the model with all three product qualities accounts for the most variance. Knowing that this model is statistically accurate at predicting machine times with product qualities we can compare the model to the LRCT metric Pella uses.

Using our final linear model and LRCT method we get:

|                        | Linear Model Estimate | LRCT Estimate | Difference |
|------------------------|-----------------------|---------------|------------|
| All Windows            | 63.3 mins             | 39.17 mins    | 24.1 mins  |
| Cardboard Covered (37) | 28.4 mins             | 24.2 mins     | 4.24 mins  |
| Plain Windows (23)     | 34.9 mins             | 39.1 mins     | 4.24 mins  |

These estimates and differences show us how important product variation is when using estimators. Additionally, though the differences between the estimates of our linear model and the LRCT became smaller once we accounted for product variation, the difference would compound through more production, leading to the conclusion that the LRCT should be used with caution.

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